OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4764**

Mechanics 4

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Thursday 16 June 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (24 marks)

1 A raindrop of mass m falls vertically from rest under gravity. Initially the mass of the raindrop is m_0. As it falls it loses mass by evaporation at a rate λm, where λ is a constant. Its motion is modelled by assuming that the evaporation produces no resultant force on the raindrop. The velocity of the raindrop is v at time t. The forces on the raindrop are its weight and a resistance force of magnitude kmv, where k is a constant.

(i) Find m in terms of m_0, λ and t. **[2]**

(ii) Write down the equation of motion of the raindrop. Solve this differential equation and hence show that $v = \frac{g}{\lambda - k}\left(e^{(\lambda - k)t} - 1\right)$. **[8]**

(iii) Find the velocity of the raindrop when it has lost half of its initial mass. **[2]**

2 A small ring of mass m can slide freely along a fixed smooth horizontal rod. A light elastic string of natural length a and stiffness k has one end attached to a point A on the rod and the other end attached to the ring. An identical elastic string has one end attached to the ring and the other end attached to a point B which is a distance a vertically above the rod and a horizontal distance $2a$ from the point A. The displacement of the ring from the vertical line through B is x, as shown in Fig. 2.

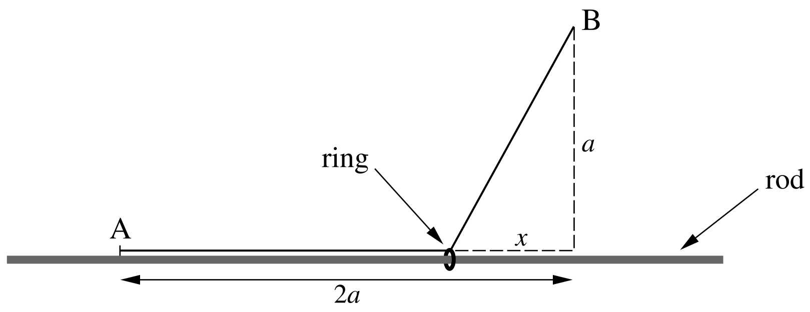

Fig. 2

(i) Find an expression for V, the potential energy of the system when $0 < x < a$, and show that

$$\frac{dV}{dx} = 2kx - ka - \frac{kax}{\sqrt{a^2 + x^2}}.$$ **[5]**

(ii) Show that $\frac{d^2V}{dx^2}$ is always positive. **[4]**

(iii) Show that there is a position of equilibrium with $\frac{1}{2}a < x < a$. State, with a reason, whether it is stable or unstable. **[3]**

Section B (48 marks)

3 A car of mass 800 kg moves horizontally in a straight line with speed $v\,\mathrm{m\,s^{-1}}$ at time t seconds. While $v \leqslant 20$, the power developed by the engine is $8v^4$ W. The total resistance force on the car is of magnitude $8v^2$ N. Initially $v = 2$ and the car is at a point O. At time t s the displacement from O is x m.

(i) Find v in terms of x and show that when $v = 20$, $x = 100 \ln 1.9$. **[10]**

(ii) Find the relationship between t and x, and show that when $v = 20$, $t \approx 19.2$. **[6]**

The driving force is removed at the instant when v reaches 20.

(iii) For the subsequent motion, find v in terms of t. Calculate t when $v = 2$. **[8]**

4 In this question you may assume without proof the standard results in *Examination Formulae and Tables (MF2)* for

- the moment of inertia of a disc about an axis through its centre perpendicular to the disc,
- the position of the centre of mass of a solid uniform cone.

Fig. 4 shows a uniform cone of radius a and height $2a$, with its axis of symmetry on the x-axis and its vertex at the origin. A thin slice through the cone parallel to the base is at a distance x from the vertex.

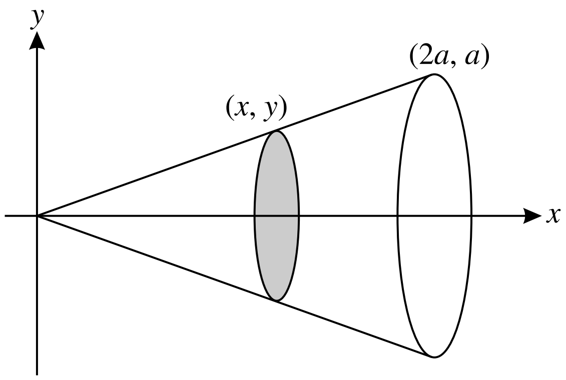

Fig. 4

The slice is taken to be a thin uniform disc of mass m.

(i) Write down the moment of inertia of the disc about the x-axis. Hence show that the moment of inertia of the disc about the y-axis is $\frac{17}{16}mx^2$. **[6]**

(ii) Hence show by integration that the moment of inertia of the cone about the y-axis is $\frac{51}{20}Ma^2$, where M is the mass of the cone. [You may assume without proof the formula for the volume of a cone.] **[8]**

The cone is now suspended so that it can rotate freely about a fixed, horizontal axis through its vertex. The axis of symmetry of the cone moves in a vertical plane perpendicular to the axis of rotation. The cone is released from rest when its axis of symmetry is at an acute angle α to the downward vertical. At time t, the angle the axis of symmetry makes with the downward vertical is θ.

(iii) Use an energy method to show that $\dot{\theta}^2 = \dfrac{20g}{17a}(\cos\theta - \cos\alpha)$. **[5]**

(iv) Hence, or otherwise, show that if α is small the cone performs approximate simple harmonic motion and find the period. **[5]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4764:** Mechanics 4

Mark Scheme for June 2011

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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1(i)	$\frac{dm}{dt} = -\lambda m \Rightarrow m = m_0 e^{-\lambda t}$	M1		
		A1		
				2
(ii)	$\frac{d}{dt}(mv) = mg - kmv$	B1	N2L	
	$\frac{dm}{dt}v + m\frac{dv}{dt} = mg - kmv$	M1	Expand derivative	
	$-\lambda mv + m\frac{dv}{dt} = mg - kmv$	M1	Substitute	
	$\frac{dv}{dt} = g + (\lambda - k)v$	A1		
	$\int \frac{dv}{g + (\lambda - k)v} = \int dt$	M1	Separate and integrate	
	$\frac{1}{\lambda - k}\ln(g + (\lambda - k)v) = t + c$	A1√		
	$g + (\lambda - k)v = Ae^{(\lambda - k)t}$			
	$v = 0, t = 0 \Rightarrow A = g$	M1	Use condition	
	$v = \frac{g}{\lambda - k}\left(e^{(\lambda - k)t} - 1\right)$ AG	E1	Convincingly shown	
				8
(iii)	$m = \frac{1}{2}m_0 \Rightarrow e^{-\lambda t} = \frac{1}{2}$	M1	Accept substituted into their expression in part (i)	
	$\Rightarrow t = \frac{1}{\lambda}\ln 2$			
	$v = \frac{g}{\lambda - k}\left(2^{\frac{\lambda - k}{\lambda}} - 1\right)$	A1	Any correct form	
				2

2(i)	$V = \frac{1}{2}k(2a - x - a)^2 + \frac{1}{2}k\left(\sqrt{a^2 + x^2} - a\right)^2$	M1	for $E = \frac{1}{2}kx^2$	
		A1		
		A1		
	$\frac{dV}{dx} = -k(a - x)$ $+k\left(\sqrt{a^2 + x^2} - a\right) \cdot 2x \cdot \frac{1}{2}(a^2 + x^2)^{-1/2}$	M1		
	$= -k(a - x) + kx\left(1 - \frac{a}{\sqrt{a^2 + x^2}}\right)$			
	$= 2kx - ka - \frac{kax}{\sqrt{a^2 + x^2}}$ AG	E1	Convincingly shown	
				5
(ii)	$\frac{d^2V}{dx^2} = 2k - \frac{ka\sqrt{a^2 + x^2} - kax \cdot x(a^2 + x^2)^{-1/2}}{a^2 + x^2}$	M1		
	$= 2k - \frac{ka^3}{(a^2 + x^2)^{3/2}}$	A1		
	$(a^2 + x^2)^{3/2} > (a^2)^{3/2} = a^3$	M1		
	$\Rightarrow \frac{ka^3}{(a^2 + x^2)^{3/2}} < k \Rightarrow V''(x) > 2k - k > 0$	E1	Convincingly shown	
				4
(iii)	$x = \frac{1}{2}a \Rightarrow V' = ka - ka - \frac{ka \cdot \frac{1}{2}a}{\sqrt{a^2 + \left(\frac{1}{2}a\right)^2}} < 0$	M1		
	$x = a \Rightarrow V' = 2ka - ka - \frac{ka^2}{\sqrt{a^2 + a^2}} = ka - \frac{ka}{\sqrt{2}} > 0$			
	Hence (as V' continuous) $V' = 0$ between $\frac{1}{2}a$ and a.	E1	Convincingly shown	
	So equilibrium. Stable as $V'' > 0$.	B1		
				3

Question	Answer	Marks	Guidance
3(i)	$800v\frac{dv}{dx} = \frac{8v^4}{v} - 8v^2$	M1	N2L with P/v
		A1	
	$\int \frac{100dv}{v^2-v} = \int dx$	M1	Separate
	$\int 100\left(\frac{1}{v-1} - \frac{1}{v}\right)dx = \int dx$	M1	Partial fractions
	$100(\ln(v-1) - \ln v) = x + c$	A1	
	$x = 0, v = 2 \Rightarrow c = -100\ln 2$	M1	Use condition
	$100\ln\left(\frac{2(v-1)}{v}\right) = x$	A1	AEF, condone m
	$v = 20 \Rightarrow x = 100\ln\left(2 \times \frac{19}{20}\right) = 100\ln 1.9$	E1	
	$\frac{2(v-1)}{v} = e^{0.01x}$		
	$2v - 2 = ve^{0.01x}$	M1	Rearrange
	$v = \frac{2}{2-e^{0.01x}}$	A1	Cao without m
		10	
(ii)	$\frac{dx}{dt} = \frac{2}{2-e^{0.01x}}$	M1	
	$\int(2 - e^{0.01x})dx = \int 2dt$	M1	Separate and integrate
	$2x - 100e^{0.01x} = 2t + c_2$	A1	
	$x = 0, t = 0 \Rightarrow c_2 = -100$	M1	Use condition
	$2x - 100e^{0.01x} = 2t - 100$	A1	Any correct form
	$x = 100\ln 1.9 \Rightarrow t \approx 19.2$ AG	E1	
		6	
(iii)	$800\frac{dv}{dt} = -8v^2$	M1	N2L
		A1	
	$\int 100v^{-2}dv = \int -1dt$	M1	Separate and integrate
	$-100v^{-1} = -t + c_3$	A1	
	$t = 19.2, v = 20 \Rightarrow -5 = -19.2 + c_3$	M1	Use condition
	$c_3 = 14.2$		
	$v = \frac{100}{t-14.2}$	M1	Rearrange
		A1	CAO
	$2 = \frac{100}{t-14.2} \Rightarrow t = 64.2$	B1	Accept $t = 45$ (time for this part of motion)
		8	

4(i)	$I_x = \frac{1}{2}my^2$	B1		
	$2I_{\text{diameter}} = I_x$	M1	Use perpendicular axes theorem	
	$I_{\text{diameter}} = \frac{1}{4}my^2$	B1		
	$I = \frac{1}{4}my^2 + mx^2$	M1	Use parallel axes theorem	
	$= m\left(\frac{1}{4}\left(\frac{1}{2}x\right)^2 + x^2\right)$	M1	Use $y = \frac{1}{2}x$	
	$= \frac{17}{16}mx^2$ AG	E1	Complete argument	
				6
(ii)	Mass of slice $\approx M\left(\frac{\pi y^2 \delta x}{\frac{1}{3}\pi a^2 \cdot 2a}\right)$	M1		
	$= \frac{3M}{2a^3}y^2\delta x$	B1	Deal correctly with mass/density	
	$I_{\text{slice}} \approx \frac{17}{16}\left(\frac{3M}{2a^3}y^2\delta x\right)x^2$	M1		
		A1		
	$= \frac{51M}{128a^3}x^4\delta x$	M1	Substitute for y	
	$I = \int_0^{2a} \frac{51M}{128a^3}x^4\,\mathrm{d}x$	M1		
	$= \frac{51M}{128a^3}\left[\frac{1}{5}x^5\right]_0^{2a}$	A1		
	$= \frac{51}{20}Ma^2$ AG	E1	Complete argument	
				8
(iii)	$\frac{1}{2}I\dot{\theta}^2 - Mg \cdot \frac{3}{2}a\cos\theta = -Mg \cdot \frac{3}{2}a\cos\alpha$	M1	Energy equation	
		B1	Position of centre of mass	
		A1	KE term	
		F1	GPE terms ft their CoM only	
	$\dot{\theta}^2 = \frac{3Mga}{I}(\cos\theta - \cos\alpha)$			
	$= \frac{20g}{17a}(\cos\theta - \cos\alpha)$	E1	Complete argument	
				5
(iv)	$2\dot{\theta}\ddot{\theta} = -\frac{20g}{17a}\sin\theta\,\dot{\theta}$	M1	Differentiate or use $I\ddot{\theta} =$ torque	
	$\ddot{\theta} = -\frac{10g}{17a}\sin\theta$	A1		
	$\approx -\frac{10g}{17a}\theta$ for small θ	M1	Use $\sin\theta \approx \theta$	
	Hence SHM	E1		
	Period $2\pi\sqrt{\frac{17a}{10g}}$	B1		
				5

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4764: Mechanics 4

General Comments

In general the performance of the candidates was very good. As in recent sessions, the standards of presentation and communication were high, though some candidates failed to include necessary detail when establishing given answers.

Comments on Individual Questions

1 (*Variable mass – evaporating raindrop)*

(i) Many candidates treated $\frac{\mathrm{d}m}{\mathrm{d}t}$ as a function of *m* instead of *t*, leading them to *m* as a linear instead of exponential function of *t*. Candidates who took the time to integrate carefully almost always found the correct expression.

(ii) This was generally not well answered. Many candidates used *F* = *ma*, treating *m* as a constant, rather than $F = \frac{\mathrm{d}}{\mathrm{d}t}(mv)$. However, most candidates separated variables and integrated accurately from their differential equation.

(iii) Nearly all candidates stated that $m = \frac{1}{2}m_0$, though some were not able to use the relationship to eliminate *t*.

2 (*Equilibrium*)

(i) This was well answered by the majority of the candidates. Some did not give sufficient detail to earn the final mark.

(ii) Most candidates earned the first two marks of this part and many took the time to simplify their expression for $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$ before considering its sign.

Very few candidates gave good attempts at showing that $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$ is positive, with many manipulating inequalities incorrectly or failing to give sufficient detail.

(iii) While most candidates knew that a position of equilibrium required $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$, many assumed that the value of *x* had to be found and attempted to solve the resulting equation, not realising that it was not easily soluble and that a change of sign approach was all that was necessary.

Almost all the candidates correctly identified the equilibrium as stable.

3 (*Variable Force*)

This question was answered well by most candidates. The use of different forms of Newton's second law for different contexts was well understood. In parts (i) and (iii) the question specifically asks for expressions for *v* in terms of *x* and *t* respectively; many candidates failed to give the required form and simply showed the given numerical answer.

(i) The application of Newton's second law was well done; very few candidates used power instead of force.

Most then proceeded to separate the variables and integrate using partial fractions and were able to show that x = 100ln1.9 accurately.

(ii) Again, this was answered well. Full marks were awarded more often here than in (i) and (iii) because the requested relationship did not require rearrangement. However, a minority of candidates found *v* in terms of *t* and could only be awarded 4 out of 6.

(iii) This was generally well answered. The change in force was understood and the resulting integral found accurately. Some candidates changed *t* to be the time after the driving force was removed; one mark was withheld in this case.

4 (*Rotation – moments of inertia and dynamics*)

(i) Most candidates gave working that led to the given answer, but the detail of the perpendicular axes theorem was often lacking.

(ii) This was not well answered by many candidates. Many candidates found the integral of $I = \frac{17}{16}mx^2$ instead of carefully considering the sum of the moments of inertia of slices of the cone. Candidates that worked in terms of ρ and substituted at the end often failed to give enough detail for a given answer; those that worked in terms of *M* having already made the substitution were more successful.

(iii) This was well answered by those candidates that attempted it, with good derivation of the various terms. Many different approaches were seen involving energy considerations.

(iv) The idea of this seemed to be understood well by the stronger candidates and the techniques applied accurately. Some interesting approaches were seen using the small angle approximation for cosine before differentiating.

GCE Mathematics (MEI)

Unit	Title		Max Mark	a	b	c	d	e	u
4751/01	(C1) MEI Introduction to Advanced Mathematics	Raw	72	55	49	43	37	32	0
		UMS	100	80	70	60	50	40	0
4752/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	53	46	39	33	27	0
		UMS	100	80	70	60	50	40	0
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	48	42	36	29	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	63	56	50	44	38	0
		UMS	100	80	70	60	50	40	0
4755/01	(FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	59	52	45	39	33	0
		UMS	100	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	55	48	42	36	30	0
		UMS	100	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	39	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01	(M1) MEI Mechanics 1	Raw	72	60	52	44	36	28	0
		UMS	100	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	64	57	51	45	39	0
		UMS	100	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	59	51	43	35	27	0
		UMS	100	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	54	47	40	33	26	0
		UMS	100	80	70	60	50	40	0
4766/01	(S1) MEI Statistics 1	Raw	72	53	45	38	31	24	0
		UMS	100	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	60	53	46	39	33	0
		UMS	100	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771/01	(D1) MEI Decision Mathematics 1	Raw	72	51	45	39	33	27	0
		UMS	100	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	58	53	48	43	39	0
		UMS	100	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	62	55	49	43	36	0
4776/02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776	(NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0